Cart Weight Data Game

Teacher Facilitation Guidelines

If you haven't done so already, please watch the short student and teacher videos, and play *Cart Weight* before or as you read these notes. Also, you should look at the student worksheet for *Cart Weight*.

Learning Goals

- Students strengthen their understanding of how linear equations can model real-world situations. (The relevant CCSSM standards are listed at the end of this document.)

Prior to Students Playing

- Set a goal for students to complete a certain level by the end of the activity. This creates some urgency and accountability for students and motivates them to focus on mastering the game and learning the underlying mathematics. In *Cart Weight*, it might be appropriate for Algebra 1 students to aim to unlock the fourth or fifth level by the end of the period. The scores needed to unlock each level are shown in the Levels dialog box on the game screen. **Note: The student worksheet for *Cart Weight* guides students only until they have unlocked the fifth level.**
- Depending on your students' experiences, consider giving them a real-world problem involving a $y = mx + b$ model prior to playing this game. For example:

  > Ramon and Jessica are competing with each other to save money to pay for the prom. Suppose Jessica starts a new savings account with no money, and then saves $10 per week in the account. Ramon starts his account with $35 and saves $5 per week. How much will they each have in their accounts after 8 weeks? What is the slope and what is the y-intercept for the equations in each of their situations?

- For suggestions on how to prepare to play Data Games with students, go to the Teacher FAQ section of the Data Games website (http://play.ccssgames.com/faq-page).

During Gameplay

- **Overall notes**
  - *Cart Weight* consists of levels of increasing mathematical challenge. Students usually try to use a guess-and-check approach for as long as it helps them succeed, rather than using the data or the math skills they have learned. The game is designed so they can succeed with this approach for the first level or two; but on the third and fourth levels, it will be difficult for them to win without analyzing the data and applying the math they have learned. The desire to perform well and advance to higher levels usually motivates them to analyze the data.
  - The student worksheet prompts students to notice that their guesses are not being displayed in the graph provided, as they might expect to see them

there. Student guesses can be seen in the table, but they should realize that it is more helpful to examine the data shown in the graph.

  - The secret teacher shortcut to unlock levels is to hold down the Option+Shift keys (Mac) or the Alt+Shift keys (Windows) while clicking the red circle next to the level you want to unlock.

- **Level 1: Dubuque**
  - On this level, the cart has no weight and each brick weighs 3, so there is a direct proportional relationship between the *number of bricks* and *total weight*. The underlying equation is the same for all games and all players: *weight* = 3 * *bricks*.
  - Most students quickly figure out the rule for this level by observing the common ratio of 3 between the total weight and the number of bricks. Students may share the "multiply by 3" rule around the classroom, but this is okay because sharing won't be helpful in all future levels!

- **Level 2: Ames**
  - The cart now has a weight, so there is no longer a direct proportional relationship. This is mentioned in the student worksheet, and students should figure this out also.
  - The underlying equation is *weight* = 4 * *bricks* + 8, which remains the same for all games and all players. Students may again share this rule around the classroom, but this sharing approach won't work by the third level.
  - Students don't yet need to click *Clear Data* between each game since the equation remains the same.
  - Some students may continue to use a guess-and-check approach over multiple games, while other students may realize how helpful it is to use **Show Movable Line**, fit the line through the data points, and get its equation.
  - The displayed equation of the linear model may include decimals even when the true underlying equation does not.

- **Level 3: Davenport**
  - The underlying equation is now different for each game and every student, so there is no longer a common rule that can be shared among students.
  - By this level, students need to have created a movable line on the graph to model the relationship between the number of bricks on the cart and the total weight of the cart. The game provides the equation of this movable line. Students may then apply this equation to determine the total weight of all future carts, given their numbers of bricks. This is similar to solving for y, given x, m, and b, in a $y = mx + b$ equation.
  - The underlying equations still involve only whole numbers for both the slope and y-intercept, so all data for *total weight* are given as whole numbers. The displayed equation of the movable line may, of course, include decimals.
  - Make sure that all students click *Clear Data* between every game, so they can focus only on the data from their current game. They are only prompted to clear their data the first time it's needed (when they move from the first to the second level), hopefully they will figure out for themselves the importance of

using this action when they have advanced to the higher levels. Alternatively, they can select a row of the table to see only that game's data highlighted in the graph.

- **Level 4: Urbandale**
    - The underlying linear equation and the weights now include decimal values, so students should figure out that rounding doesn't result in the best scores. They will need to grapple with using non-whole numbers.
- **Level 5: Waterloo**
    - The game now introduces some variability to the data. While there is still one underlying linear equation, not all data points lie exactly on it. Students will need to keep making adjustments to find a line of best fit for the data points.
- **Level 6: Minot**
    - This level is a complex and different activity. **Minot has its own Teacher Notes, which you can find beginning on page 5 of this document.**
- **Possible Extensions**
    - Discuss with students why on the second level and above in *Cart Weight*, it takes two wild guesses to determine the line. In the Data Games *Proximity* and *Shuffleboard*, it takes only one wild guess. The reason is that in the other games, you can determine the y-intercept, or starting position, before you begin play, so you already know a point on the line. The underlying mathematical axiom is the same in each game: "Two points determine a line."
    - Reinforce students' Algebra skills by asking how they could come up with the equation of the movable line if the computer didn't provide it.

Answers to Student Worksheet Questions

- "(Q1) What do you think is the highest possible score in each game?" – **500**
- "(Q2) Are your guesses or scores shown in the graph? (yes/no)" – **No**
- "(Q3) Describe briefly how playing on this second level is different than playing on the first level:" – **In teacher language: On the Ames level, because the cart has a weight, there is a positive y-intercept and no direct proportion, so the rule is a little more complex.**
- "(Q4) Looking at the data points for the most recent game on the graph, what type of relationship or function does there seem to be between the number of bricks and the total weight of the cart?" – **Linear**
- "(Q5) In the game you just finished playing, how much does each brick weigh?" – **Answers will vary. This just prompts students to think about the meaning of the slope.**
- "(Q6) In the game you just finished playing, how much would a cart weigh that had 0 bricks in it?" – **Answers will vary. This just prompts students to think about the meaning of the y-intercept.**
- "(Q7) What does the slope of your movable line represent in this game? Explain." – **The weight of each brick. Each new brick adds this much weight.**

- "(Q8) What does the *y*-intercept of your movable line represent in this game? Explain." – **The weight of the empty cart. When there are no bricks on the cart, it weighs this much.**
- "(Q9) Suppose you were asked to help out a friend who hadn't yet played how to win this game on the fourth level. Explain below, as if he or she were just starting it on his/her own computer." – **Explanations will vary, but should include the steps of making two guesses, fitting a movable line through the two data points, and using the equation to substitute in values for *bricks* to find *weight*.**
- "(Q10) If the number of bricks doubles from one cart to the next, would the total weight of the cart double also? Explain why or why not. Give an example from your data if you can." – **No, because the cart has a weight that doesn't double. There is no longer a direct proportional relationship after the first level, so the positive *y*-intercept makes this not true.**

Challenges Introduced on Each Level

Level	Positive *y*-intercept (cart has weight)	Equation changes with each game	Non-whole numbers	Variability (not all points lie exactly on line)	Two sizes of bricks
1: Dubuque					
2: Ames	⊗				
3: Davenport	⊗	⊗			
4: Urbandale	⊗	⊗	⊗		
5: Waterloo	⊗	⊗	⊗	⊗	
6: Minot	⊗	⊗			⊗

Relevant Common Core State Standards for Mathematics

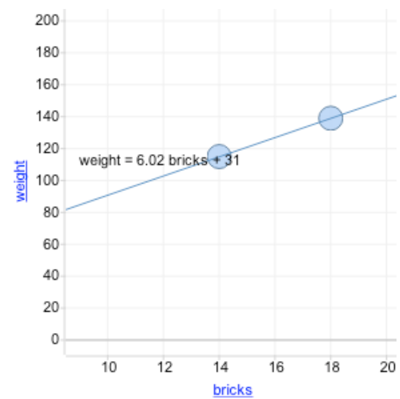
- Analyze proportional relationships and use them to solve real-world and mathematical problems (6.RP.1, 6.RP.2, 6.RP.3, 7.RP.1, 7.RP.2)
- Understand the connections between proportional relationships, lines, and linear equations. (8.EE.5)
- Use functions to model relationships between quantities (8.F.4)
- Investigate patterns of association in bivariate data (8.SP.1 – describe linear association; 8.SP.2—fit a line to data; 8.SP.3—interpret slope and intercept in linear model to solve problems)
- Solve systems of linear equations exactly and approximately (A.REI.6)
- Represent and solve equations and inequalities graphically (A.REI.10)
- Interpret functions that arise in applications in terms of the context (F.IF.4)
- Interpret expressions for functions in terms of the situation they model (F.LE.5)

- Summarize, represent, and interpret data on two categorical and quantitative variables (S.ID.6)
- Interpret linear models (S.ID.7)
- Make sense of problems and persevere in solving them (Standard for Mathematical Practice - 1)
- Model with mathematics (Standard for Mathematical Practice - 4)
- Use appropriate tools strategically (Standard for Mathematical Practice - 5)
- Attend to precision (Standard for Mathematical Practice – 6)

Teacher Notes for Minot Level of *Cart Weight*

What's the best way to analyze the data in the Minot level of *Cart Weight*? That's not such an easy question.

In Data Games, we generally assume students will make a graph of the data, put a line on it, and use the formula for the line to make predictions. But it's not the only way—and that's a good thing. Your students can approach the problem in different ways, and furthermore, you can return to the problem to highlight different strategies. Let's see this in practice. We're in the third level, Davenport:



The Graph Strategy
Take two measurements just to get data, then put a line through the two points. This will give you an equation in *bricks* from which you can calculate the weight of the cart.

In the illustration, the formula reads

$$weight = 6.02\ bricks + 31$$

So if the next cart has 11 bricks, we can plug in the 11 and (assuming 6.02 is really 6) get 97 for the weight.

The Table Strategy
If you look at the table instead, you can reason about the bricks from the numbers directly. Here's the table for the first two points:

bricks	weight
14	115
18	139

A student might reason, "the difference between the two measurements is 4 bricks, and that's 24 pounds, so it's 6 pounds per brick." Then when we predict for 11 bricks, we subtract three bricks (18 pounds) from the 115 for 14 bricks, to give us 97.

The Algebra Strategy
You could use algebra directly as well. We have two data points; convert them to equations in two variables and solve the system. Let C be the weight of the cart and B be the weight of one brick. Then

$$C + 18B = 139$$

$$C + 14B = 115$$

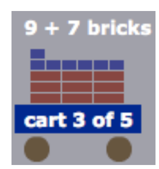
Now you subtract the two equations to eliminate C and get $4B = 24$; substitute in to find $C = 31$, and now you can construct the general equation,

$$weight = 31 + 6\ bricks.$$

Now we can use this equation as before: plug in 11 for *bricks* and find the weight - 97.

Which is best?
From the point of view of the traditional math curriculum, the algebra strategy, if not best, at least "smells" more like real math. You do algebra! You have multiple equations! And it works. But let me make a couple of points:

- If the data are not perfect—if there's variability, as in real life or in the Waterloo level—it's not clear how to adjust for new data.
- It's not obvious to most students how to set up those two equations—where the variables are the weight of the cart and the weight of one brick—and then convert the parameters you solve for into what you really need, which is a way of calculating the weight.
- In contrast, the graph strategy immediately gives you an equation for what you want to know. It's a tradeoff, however: to really understand what's going on, students need to figure out the meaning of the slope and intercept (which are the same two parameters as in the algebra strategy), and there is some slop in the positioning of the line: it's easy to be off by a few pounds.
- The graph strategy is also more concrete—or perhaps usefully abstract: as more carts cross the screen, it gives you a growing picture of the data set. Although some people really grok algebraic expressions, for most, the graph tells the story better.
- On the other (original) hand, the algebra-strategy equations you get here are simple to solve. Because the coefficient for the cart is always "1," you can always eliminate one variable easily.



What about Minot?
In the Minot level, however, life gets harder. There are two weights of bricks.

If we had a 3-D graph and a movable plane, we could handle this—but we don't. What strategies can we use? Guess and check works, but it can be frustrating. Could it be that algebra is best?

Here we have three points from the table:[1]

bricks	weight	guess	score	smBricks
4	70	30	0	2
6	100	140	0	6
9	140	200	0	7

We can use B for the weight of each big brick, S for the weight of each small brick, and C for the empty cart weight:

$$C + 9B + 7S = 140$$

[1] We are glossing over a very important issue: how many data points do you need? In many of these games, that's a crucial question. Students must learn to ask this question, and recognize how many independent unknowns they need in order to describe the system.

$$C + 6B + 6S = 100$$

$$C + 4B + 2S = 70$$

Although this is a system of three equations and three unknowns, the naked "C"s make it easier. Algebra 2 students can use elimination and repeated substitution to solve the system. If you go as far as using matrices to solve the system as a whole, that works as well.

This could be a good lesson, but it's hardly in the original spirit of Data Games. Picture the flow: students make three moves and then retire for maybe 15 minutes to solve a hairy algebra problem before returning to make their final two moves. If they did it right, those moves could be really satisfying, but still: not much like a game.

Dynamic Data Approach to Minot

Surely there's a more dynamic, data-oriented way to approach this—without a 3-D graph.

First let's notice something obvious but important: in the earlier levels, for each game, all the points lay in a straight line.

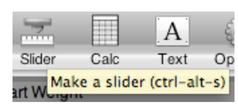
Now something not so obvious: if we knew the weight of the small bricks, we could subtract that from the total. If we plotted that result instead of the total weight, we'd get a straight line too. It would be just like the problem of having one size of brick. (Notice that this is right out of Pólya: make it into a simpler problem.)

How shall we do this? We don't know the weight of the small bricks.

Here is the dynamic-data-oriented strategy: let's pretend we do.

We'll make a slider for the weight of the small bricks, and make the graph assuming that the slider is correct. We'll see the shape of the graph. All we have to do is vary the slider until the points line up!

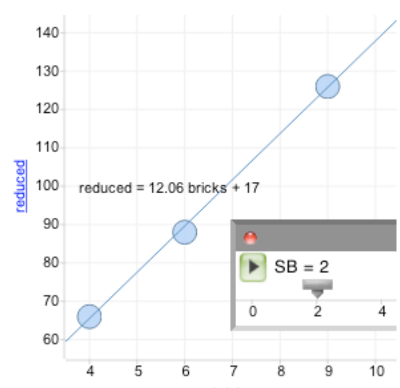
Need step-by-step instructions? Let's try it:



- Start a game at the Minot level and collect at least three data points.
- Make a slider by clicking the slider in the tool bar (shown at right).
- Change its name from *v1* to *SB* for "small brick weight."

Now we need to make an attribute for the reduced weight, and give it a formula.

- In the table, choose **New Attribute in Carts** from the Gear menu. (We want to compute this for each cart. That's why the new attribute is in Carts and not in Games.)
- Give it a name (such as *reduced*) and a formula. I used *weight – SB * smBricks*. Be sure you understand that formula!

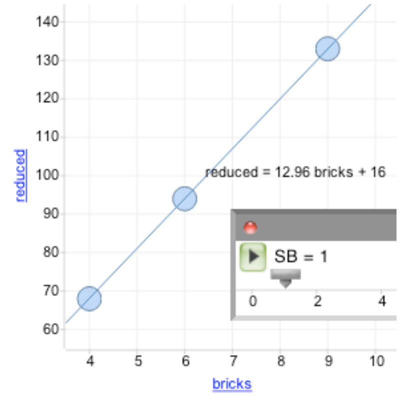


Now we use it in the graph.

- Drag *reduced* to the vertical axis of your graph, replacing *weight*, so it's now *reduced* against *bricks*.

- Move the slider! (or edit the value) At right, we try *SB* = 2. You can see that the three points do not line up. The middle one is low compared with the line.
- See which way makes the miss better and which is worse. In this case, setting *SB* to 3 makes it worse, so we try *SB* = 1. It looks pretty good (illustration lower right).[2]

With the equation from the picture, *reduced* = 12.96 *bricks* + 16, we suspect that the small bricks weigh 1, the big bricks 13, and the cart weighs 16. This turns out to be correct.



This is only one way to approach this without solving the 3 x 3 set of equations. Notice that it still requires substantial thinking and understanding on the part of the students—just less algebra.

[2] It happens that in this level the weights are whole numbers. Students wouldn't necessarily know that, but the same procedure applies: just slide until the points line up. With non-integers, judging that will be harder.